wizPROF 2024

THURSDAY MARCH 21ST 2024

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WERELDWIJDE WISKUNDE WEDSTRIJD

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GOOD LUCK AND MOST OF ALL HAVE FUN !

- calculators are not allowed
- you may use 75 minutes
- only a pencil, an eraser and scribbling paper are allowed
- results and prizes will arrive at school at the end of May
- answers will be posted on the website about March 29th
- solutions will be posted on the website about April 20th

wizPROF
havo 4 & 5
vwo 3, 4, 5 & 6

1. What is the result of $\dfrac{2 \times 0.24}{20 \times 2.4}$?

A. 0.01   **B.** 0.1   **C.** 1   **D.** 10   **E.** 100

2. Which square is split up into two pieces that do **not** have the same shape?
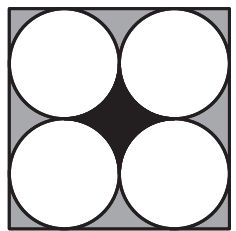

A.   **B.**   **C.**   **D.**   **E.**
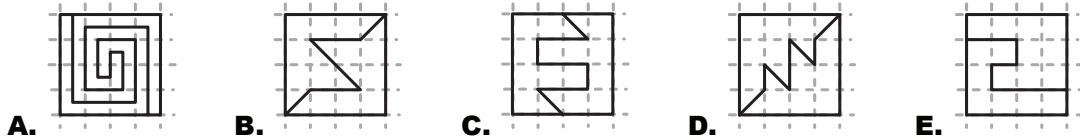
3. The number of dots on opposite faces of a die add to 7.
Vertex *P* is adjacent to the faces with 1, 2 and 3 dots and has 'vertex sum' 6, namely 1 + 2 + 3 = 6
We look at the vertex sum of each of the vertices *Q*, *R* and *S*.
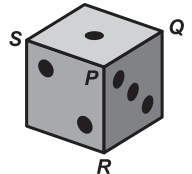
What is the largest sum?

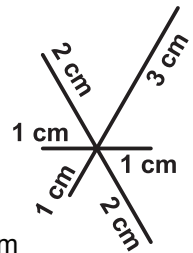**A.** 7   **B.** 9   **C.** 10   **D.** 11   **E.** 15

4. The image shows a square-shaped mosaic.
The 4 identical circles touch each other and the square.

What is the ratio between the black and the grey area?

A. 1:4   **B.** 1:π **C.** 1:3   **D.** 2:3   **E.** 3:4

5. *Tim* wants to draw the figure shown, without lifting his pencil from the paper.
The length of each of the line segments is shown next to it.

What is the shortest total distance he has to draw with his pencil if he is allowed to choose where he starts?

**A.** 14 cm   **B.** 15 cm   **C.** 16 cm   **D.** 17 cm   **E.** 18 cm

6. A hopping game is played in the following way: you jump from square to square,
always alternating between left foot - both feet - right foot - both feet - left foot - both feet,
and so on.
Maya played the game starting with her left foot on the square with number 1 (see picture).
After that she jumped exactly 2023 times.

In how many squares did her left foot touch the ground?

A. 506   **B.** 1012   **C.** 1518   **D.** 2024   **E.** 4048

7. *John* makes a structure out of cubes.
He does this by sticking five cubes to the five visible faces of a cube, as seen below.
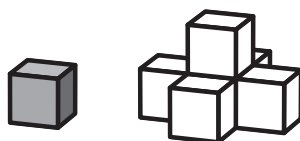

The next structure again is created by sticking cubes to the visible faces.

How many cubes are needed for this step?

A. 8   **B.** 9   **C.** 10   **D.** 13   **E.** 19

8. A three-digit palindrome has the form 'aba', where the digits a and b can either be the same or different.

What is the sum of the digits of the largest three-digit palindrome that is a multiple of 6?

A. 16   **B.** 18   **C.** 20   **D.** 21   **E.** 24

9. We draw a square *ABCD* and a regular hexagon with side *OB*, where *O* is the centre of the square.
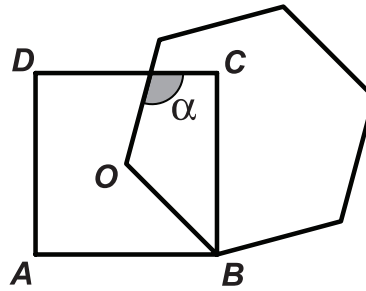

What is the size of angle α (see picture)?

A. 105°   **B.** 110°   **C.** 115°   **D.** 120°   **E.** 125°

10. *Ardal* builds a 40-meter fence around a rectangular field.
The side lengths of the field are both prime numbers.

What is the maximum area of the field in m² ?

A. 51   **B.** 84   **C.** 91   **D.** 96   **E.** 99

11. A rectangle is divided into three regions of equal area.
One of the regions is an equilateral triangle, with side length 4 cm,
the other two are trapezia, as shown in the figure.
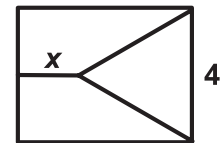
What is, in cm, the length of *x*?

A. $\sqrt{2}$ **B.** $\sqrt{3}$ **C.** $2\sqrt{2}$ **D.** 3   **E.** $2\sqrt{3}$

12. *Jelena* places capital letters *A*, *B*, *C* and *D* in the 2 × 4 table as shown below.
She wants to make sure that in each of the two rows as well as in each of the three 2 × 2 squares each letter appears only once

In how many ways can she do this?

A. 12   **B.** 24   **C.** 48   **D.** 96   **E.** 198

13. *Sanjay* cuts out three circles from three different pieces of coloured cards.
He places them on top of each other, as shown in figure 1.
He then moves the circles so that all three circles are tangent to each other,
as shown in figure 2.
In figure 1, the area of the visible black region is seven times the area of
the white circle.

figure 1   figure 2

What is the ratio between the areas of the visible black regions in the figures?

A. 3:1   **B.** 4:3   **C.** 6:5   **D.** 7:6   **E.** 9:7

14. *Mary's* daughter gave birth to a baby girl today.
In two years' time, the product of the ages of *Mary*, her daughter and granddaughter will be 2024.
The ages of *Mary* and her daughter are even numbers.

How old is *Maria* now?

**A.** 42   **B.** 44   **C.** 46   **D.** 48   **E.** 50

15. Inside an equilateral triangle, a point *P* is chosen.
From P we draw three segments parallel to the sides of the triangle, as shown in the figure.
The lengths of the segments are 2, 3 and 6.
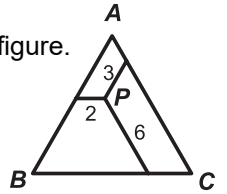
What is the perimeter of the triangle?

A. 22 **B.** 26 **C.** 33 **D.** 39 **E.** 44

16. In the diagram alongside, a circle is drawn on each of the twelve vertices in which a number is written.
The number inside each square indicates the product of the numbers at its four vertices.
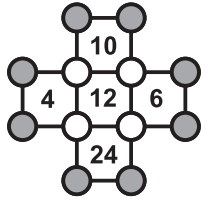
What is the product of the numbers in the eight grey circles?

A. 20 **B.** 40 **C.** 80 **D.** 120 **E.** 480

17. On the table there are four vases.
In vase 1 there are as many flowers as the number of vases containing 1 flower.
Vase 2 contains as many flowers as the number of vases containing 2 flowers.
Vase 3 contains as many flowers as the number of vases containing 3 flowers.
Vase 4 contains as many flowers as the number of vases without a flower.

How many flowers are in all the vases together?

A. 2 **B.** 3 **C.** 4 **D.** 5 **E.** 6

18. *Hans* has n^3 small equal cubes.
With these he made a large cube and painted the entire outer surface of it.
The number of small cubes with only one face painted is equal to the number of unpainted cubes.
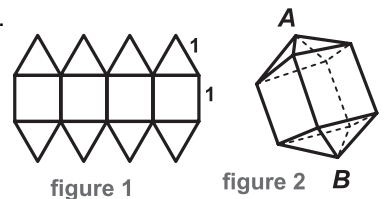
What is the value of *n*?

A. 4 **B.** 6 **C.** 7 **D.** 8 **E.** 10

19. *Christina* has a set of cards numbered 1 to 12.
She places eight of them at the corners of an octagon in such a way that the sum of each pair of numbers which share a common edge is a multiple of 3.

Which numbers did *Christina* **not** use?

A. 1, 5, 9 and 12 **B.** 3, 5, 7 and 9 **C.** 1, 2, 11 and 12 **D.** 5, 6, 7 and 8 **E.** 3, 6, 9 and 12

20. *Otis* has a net consisting of squares and triangles, as shown in figure 1.
All the side lenghts are 1 cm long. He folds the net up into a shape as shown in figure 2.

What is the distance, in cm, between vertices *A* and *B*?

A. $\sqrt{5}$ **B.** $1+\sqrt{2}$ **C.** $\dfrac{5}{2}$ **D.** $1+\sqrt{3}$ **E.** $2\sqrt{2}$

21. The integers 1 t/m *n* are multiplied by each other $(1\cdot2\cdot3\cdot \ldots \cdot n)$.
The result is written as the product of prime numbers in ascending order.
In the picture this is done for a certain value of *n*.
You can see that 47 is the highest prime number and that the prime number 13 occurs four times.

Which exponent of 17 is covered by ink?

$$2\cdot3\cdot5\cdot7\cdot11\cdot13^4\cdot17\ldots\cdot43\cdot47$$
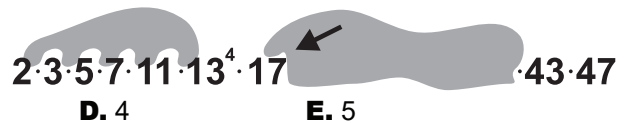
A. 1 **B.** 2 **C.** 3 **D.** 4 **E.** 5

22. The sum of the digits of the number *n* is three times the sum of the digits of the number *n* +1.

What is the smallest possible sum of the digits of *n*?

A. 3 **B.** 9 **C.** 12 **D.** 15 **E.** 27

23. *Carl* tells the truth one day and lies the next day. He alternates this every day.
One day, he made exactly four of the five statements A, B, C, D and E.

Which statement could he **not** have made on that day?

A. I lied yesterday and I will lie tomorrow.
B. I'm telling the truth today and I will tell the truth tomorrow.
C. 2024 is divisible by 11.
D. Yesterday was Wednesday.
E. Tomorrow will be Saturday.

24. *Jill* has a number of all-black, all-grey and all-white 1 × 1 × 1 cubes.
She uses 27 of them to build a large 3 × 3 × 3 cube.
She wants the surface of this large cube to be exactly one-third black, one-third grey and one-third white.

What is the difference between the largest and smallest number of black cubes that can be used?

A. 1   **B.** 3   **C.** 6   **D.** 7   **E.** 9

25. *Ann* has rolled a normal die 24 times. All numbers from 1 to 6 has been thrown at least once.
The number 1 has been thrown more times than any other number.
Ann added up all the numbers thrown.

What is the maximum sum of the numbers she could have thrown?

A. 83   **B.** 84   **C.** 89   **D.** 90   **E.** 100

26. *Olya* was walking through the park. She walked half of the total time at a speed of 2 km/h.
She walked half of the total distance at a speed of 3 km/h.
The rest of the time she walked at a speed of 4 km/h.

What part of the total time did she walk at a speed of 4 km/h?

A. $\frac{1}{17}$ **B.** $\frac{1}{15}$ **C.** $\frac{1}{14}$ **D.** $\frac{1}{12}$ **E.** $\frac{1}{4}$

27. *Ali* divided the integers from 1 to 25 into two groups.
He then removed some of the integers so that the products of the integers in each group are equal.

What is the minimum number of integers *Ali* could remove?

A. 4   **B.** 5   **C.** 6   **D.** 7   **E.** 8

28. Twenty points are equally spaced on the circumference of a circle.
Between pair of points, line segments are drawn that are longer than the radius and shorter than the diameter of the circle.
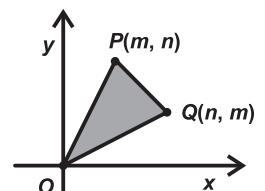
How many of these line segments can we draw?

A. 90   **B.** 100   **C.** 120   **D.** 140   **E.** 160

29. Suppose *m* and *n* are integers with $0 < m < n$.
Let $P = (m, n)$, $Q = (n, m)$ and $O = (0, 0)$.

For how many pairs *m* and *n* is the area of triangle *OPQ* equal to 2024?

A. 4   **B.** 6   **C.** 8   **D.** 10   **E.** 12

30. Each line in the square *ABCD* with side length 5 divides the area of the square in the ratio 2:3.
The lines form two pairs of parallel lines, with $CL = DK = 1$.

What is the area of quadrilateral *EFGH*?

A. 24/29   **B.** 25/29   **C.** 26/29   **D.** 27/29   **E.** 28/29