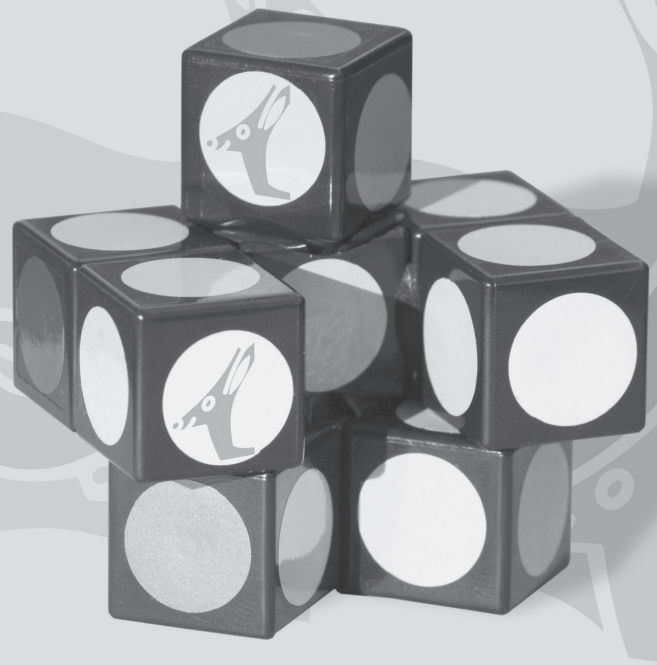
wizPROF 2013

wereldwijde wiskundewedstrijd

W4Kangoeroe

www.w4kangoeroe.nl

March 21st 2013



Good luck and most of all have fun.

calculators are not allowed

you may use 75 minutes

scribbling paper is allowed

results and prizes will arrive at school in May

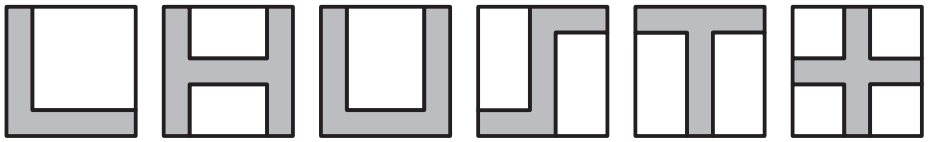
answers will be posted on the website March 28th

solutions will be posted on the website April 19th

wizPROF
havo 4 & 5
vwo 3, 4, 5 & 6

wizPROF 2013

1. *Eve* has drawn figures on square sheets of paper:



Some of these figures have the same perimeter as the whole piece of paper.
How many figures?

A. 2 **B.** 3 **C.** 4 **D.** 5 **E.** 6

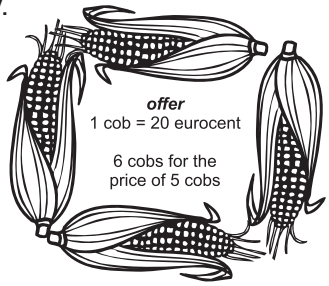
---

2. By which of the following numbers can the outcome of 200013 − 2013 not be divided?

A. 2 **B.** 3 **C.** 5 **D.** 7 **E.** 11

---

3. Mrs *Farmer* buys 4 corncobs for each of the four persons of her family.
The shop has a special deal, see the picture alongside.



How many euro does she have to pay?

A. € 0,80 **B.** € 1,20 **C.** € 2,80 **D.** € 3 **E.** € 3,20

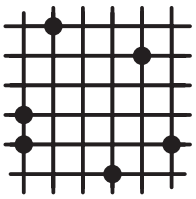
---

4. *Charles* has multiplied three of the numbers 2, 4, 16, 25, 50 and 125 together.
The outcome he got was 1000.
What is the sum of the three numbers?

A. 70 **B.** 77 **C.** 91 **D.** 131 **E.** 143

---

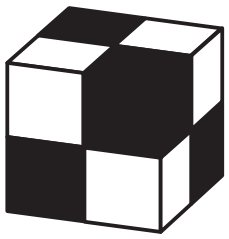
5. Six dots have been drawn on a sheet of squared paper. The squares are 1 by 1. We draw triangles with three of these six dots as vertices and compute their area.
What is the smallest area you can get?

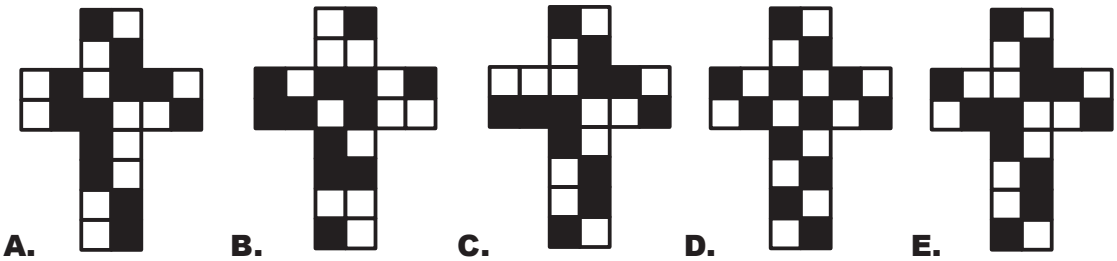


A. $\frac{1}{2}$ **B.** 1 **C.** $1\frac{1}{2}$ **D.** 2 **E.** $2\frac{1}{2}$

---

6. On the faces of a cube, black and white squares are painted.
This makes it look as if the cube is made out of four black and four white smaller cubes. See the picture.
Which of the following nets belongs to this cube?





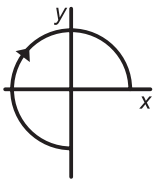
A. **B.** **C.** **D.** **E.**

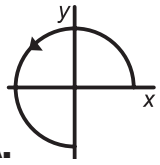
---

7. We consider all three-digit numbers that are a positive multiple of 4.
What is the difference between the largest and the smallest of these numbers?

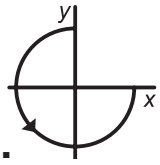
A. 224 **B.** 225 **C.** 896 **D.** 899 **E.** 900

8. We look at a three-quarter circle with the origin as centre, on which a direction is indicated; see the picture. We first rotate the whole counter-clockwise over 90° and next reflect in the x-axis.
What is the result?

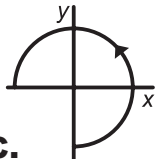




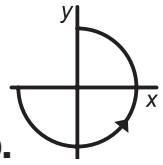
A.



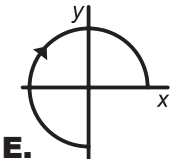
B.



C.



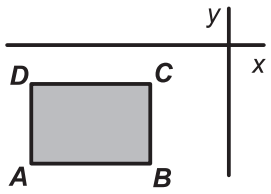
D.



E.

---

9. Rectangle *ABCD* has been drawn in a plane with coordinate axes. The sides are parallel to the axes. The rectangle is below the *x*-axis and to the left of the *y*-axis, as shown in the picture. For each of the points *A*, *B*, *C* and *D* we divide the *y*-coordinate by the *x*-coordinate. For which point will the outcome have the smallest value?



A. *A* **B.** *B* **C.** *C* **D.** *D*
E. depends on the rectangle

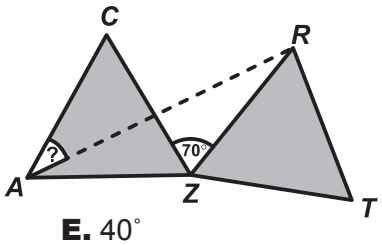
---

10. Which of the following numbers is the largest?

A. $\sqrt{20}\cdot\sqrt{13}$ **B.** $\sqrt{20}\cdot 13$ **C.** $20\cdot\sqrt{13}$ **D.** $\sqrt{201}\cdot 3$ **E.** $\sqrt{2013}$

---

11. The equilateral triangle *AZC* is rotated around *Z* to triangle *RZT*. Here $\angle CZR = 70°$.
How large is the angle $\angle CAR$?



A. 20° **B.** 25° **C.** 30° **D.** 35° **E.** 40°

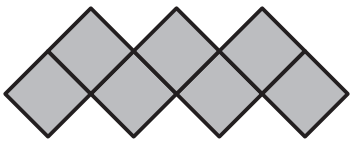
---

12. Four buttons are aligned in a row, as in the figure alongside. When you press a button, the face of that button changes, as do those of the faces immediately next to it:
a happy face turns sad and a sad face becomes happy.
You would like to get all faces happy.
How often do you have to press for that?

A. 2 **B.** 3 **C.** 4 **D.** 5 **E.** 6

---

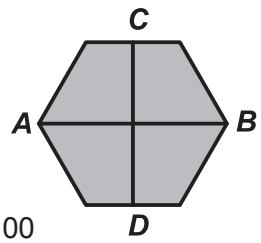
13. The figure alongside consists of a "zigzag" of seven squares of 1 x 1 cm. Its perimeter is 16 cm.
How many cm is the perimeter of a "zigzag" that is constructed similarly out of 2013 squares?



A. 2022 **B.** 4028 **C.** 4032 **D.** 6038 **E.** 8050

---

14. The line segment *AB* connects two opposite vertices of a regular hexagon. The line segment *CD* connects the midpoints of two opposite sides. The area of the hexagon is 60.
What is the product of the lengths of *AB* and *CD*?



A. 40 **B.** 50 **C.** 60 **D.** 80 **E.** 100

---

15. After a test it turned out that the average score would have been 1.2 higher if every boy would have scored 3 more points.
What percentage of the class was a girl?

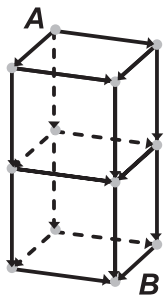
A. 20% **B.** 30% **C.** 40% **D.** 50% **E.** 60%

16. *Charles* makes sequences of five consecutive positive whole numbers with the property: the sum of three of these numbers equals the sum of the other two.
How many of such sequences do exist?

A. 0 **B.** 1 **C.** 2 **D.** 3 **E.** more than 3

---

17. How many different shortest routes are there from *A* to *B*?



A. 6 **B.** 8 **C.** 9 **D.** 12 **E.** 15

---

18. For a six-digit number the following is known: the sum of the digits is even and the product is odd.
Which of the following statements is true?

A. two or four of the digits are even
B. the number of odd digits is odd
C. such a number does not exist
D. the digits could all be different
E. A, B, C and D are all false

---

19. 2013 people live on an island. Some are knights, they always speak the truth. The others always lie. Every day one of the people says: ``When I have left, the number of knights will equal the number of liars.'' After that he leaves the island.
After 2013 days the island is uninhabited.
How many liars were there at the beginning?

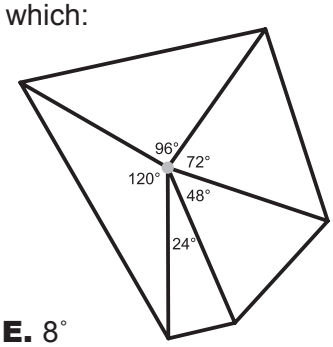
A. 0 **B.** 1006 **C.** 1007 **D.** 2013 **E.** you cannot tell

---

20. We would like to put together a closed circle of isosceles triangles, for which:

- the tops are the same point,
- the smallest top angle is a whole number of degrees and
- the other top angles are consecutive multiples of the smallest top angle.

The picture shows an example.
We want a circle with as many isosceles triangles as possible.
What is the smallest top angle in that case?



A. 1° **B.** 2° **C.** 3° **D.** 6° **E.** 8°

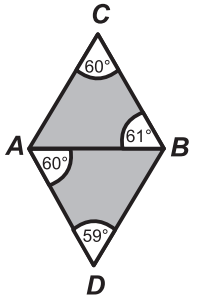
---

21. Procedure ''changesum'' makes a new group out of a group of three numbers. Each number is replaced by the sum of the other two. This way, the group {3, 4, 6} is changed by ''changesum'' into the group {10, 9, 7}. The new group is changed by ''changesum'' into {16, 17, 19}. Now we are going to change the group {1, 2, 3} a number of times using ''changesum''.
How many times do we have to do that to get the number 2013 into the group?

A. 8 **B.** 9 **C.** 10
D. 2013 appears more than once in the group **E.** 2013 will never appear in the group

---

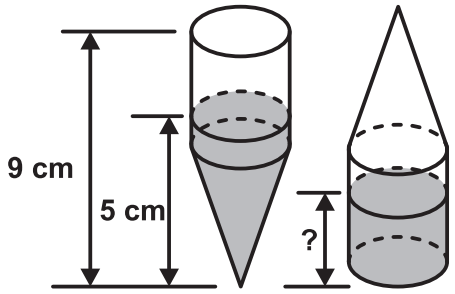
22. *Fred* had meant to draw a rhombus by joining two equilateral triangles. Unfortunately, he did not draw correctly all lengths.
It turns out that the angles are as shown in the picture.
Which of the following five line segments in *Fred*'s drawing is the longest?



A. *AB* **B.** *AC* **C.** *AD* **D.** *BC* **E.** *BD*

23. A container of 9 cm height is built up from a cilinder and a cone.The cone is less than 5 cm high.
The container is filled with water for $\frac{1}{3}$ part.
If the container is held with the cone down, the water level is 5 cm high.

How high will the water level be if we turn the container upside down?



A. 1,5 cm **B.** 2 cm **C.** 2,5 cm **D.** 3 cm **E.** 3,5 cm

---

24. One day in 2013, *Charles* and his son Fred notice:
if you multiply our ages, you get 2013.
In what year was *Charles* born?

A. 1952 **B.** 1953 **C.** 1980 **D.** 1981 **E.** 1982

---

25. We write the number $\frac{1}{1024}$ as decimal number.
You can always write 0's after a decimal fraction - for example extend 0.307 to 0.307000 - but we will not do that in this question.
How many digits will there be after the decimal point?

A. 7 **B.** 8 **C.** 9 **D.** 10 **E.** 1024

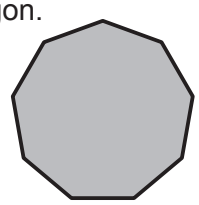
---

26. *Fred* writes down eleven fractions. He uses each of the numbers 1 through 22 once, either in a numerator or in a denominator.
What is the largest number of fractions that are equal to a whole number?

A. 7 **B.** 8 **C.** 9 **D.** 10 **E.** 11

---

27. In a regular 9-gon we construct all triangles with as vertices vertices of the 9-gon.
On the inside of how many of these triangles will the centre of the 9-gon lie?



A. 30 **B.** 31 **C.** 32 **D.** 36 **E.** 42

---

28. In a car race, the cars drive with constant speed. They depart from the same starting point one hour apart. The first car drives with constant speed of 50 km/h. The second car departs an hour later with constant speed of 51 km/h. The third car departs an hour later again, with a constant speed of 52 km/h. And so on. The final car departs 50 hours after the first with a constant speed of 100 km/h.
What is the speed of the car that is in the lead after 100 hours from the start of the first car?

A. 50 km/h **B.** 66 km/h **C.** 75 km/h **D.** 84 km/h **E.** 100 km/h

---

29. A gardener has to plant 100 trees along a path: oaks and lime trees.
Between two oak trees there cannot be exactly five trees.
What is the largest number of oak trees the gardener could possibly plant?

A. 15 **B.** 30 **C.** 48 **D.** 50 **E.** 52

---

30. *Dana* is walking along a street and sees a tractor pull an uprooted tree.
She decides to measure the length of the tree. Walking in the direction the tractor is driving, she passes the tree in 140 steps. Walking the opposite way, she passes the tree in 20 steps.
Dana takes steps of 1 meter.
How long is the tree in meters?

A. 30 **B.** 35 **C.** 40 **D.** 48 **E.** 80